Airlines Revenue Management

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Introduction

- Upt to 1978 the american Air Transport was regulated by Civil Areonautics Board, with high and almost fixed prices.
- After the deregulation, a lot of low-cost company arised, lowering the ticket prices and working on the reduction of expenses and services.
- This brings the importance of the Revenue Management (RM), technique of price and classes management in order to maximize revenues.

Revenue Management can be applied under these conditions:

- We are selling a fix quantity Si sta vendendo una fissata quantità;
- The customers books beforehand;
- Seller has a set of charges, each of them with a fixed price (at least in the short term)
- Seller can change the availability of classes over time

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- Optimization techniques to determine the optimal limit of booking for each class;

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Measurement of efficiency of RM

- Before the deregulation, a air travel was considered efficienty if it travel with at least 75% of capacity
- In the $'80s$ people tried to maximize the CASM "Cost per available seat mile", but it brought to an excessive raise of ticket prices
- A compromize between them is a RAMS="revenue per available seat mile ", that it is still used by airlines
- Weak points: it doesn't depend on distribution of customers on different classes and it does not consider expenses

	Average between six biggest airlines	Southwest Airlines	JetBlue	AirTran	Average between low cost airlines
RAMS	**10.5**	**8.4**	**7.7**	**9.1**	**8.4**
CAMS	11.1	7.5	6.3	8.5	7.4
NET	-0.6	0.9	1.4	0.6	1.0

We are going to analyze three different situations:

- **Capacity assignment** : Supposing to have different classes, how many seats we reserve for the highest price class? How many tickets we decide to sell for the lower classes?
- **Network Management**: How can we manage booking in the case of multiple routes?
- **Overbooking**: How many booking we accept over the capacity of the plane, in order to optimize revenue and protect us from cancelation and no-show?

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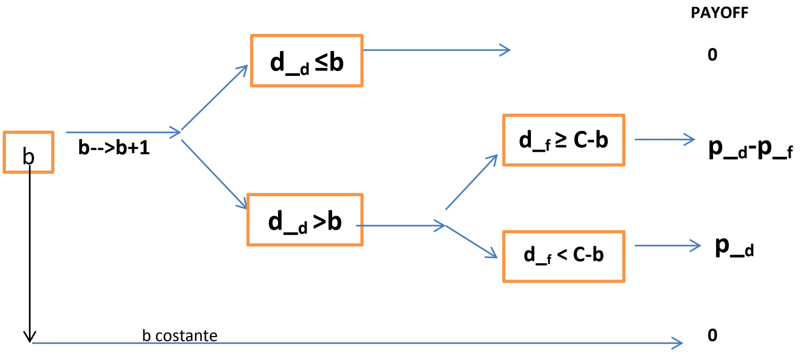
We suppose customers who buy at the lower fare book before the customers who buy at the higher fare
We want to decide how many seats assign to the former, knowing that we will have a random request from the customers who buy at the higher fare.
Two classes problem:

- There are customers who buy at a lower fare p_d, the request for this fare is d_d
- There are customers who buy at a higher fare p_f, the request for this fare is d_f
- C is the capacity of the plane
- $F_d(x) = P(d_d \leq x)$
- $F_f(x) = P(d_f \leq x)$
  The aleatority is given by d_d and d_f

Supposing we decide a limit b on the tickets with a lower fare What happen if we raise this limit to $b + 1$?



Defining $h(b)$ the function that, fixed b seats with a discount fare, make the calculation of the expected revenue in the case we want to add one seat for the lower fare, we obtain:

$$E[h(b)] = p_d(1 - F_d(b))(F_f(C - b)) + (p_d - p_f)(1 - F_d(b))(1 - F_f(C - b))$$

$$= [1 - F_d(b)](p_d - p_f + p_f F_f(C - b))$$

If the expected value of this operation is positive, than we should raise of one the number of tickets with a lower fare, otherwise we should not. The first factor is always positive, so the sign depend on the second factor. Summarizing:

- If $p_d \leq p_f(1 - F_f(C - b))$ we don't change anything;
- If $p_d \geq p_f(1 - F_f(C - b))$, then we will raise of one the number of tickets with a lower fare.

We could iterate this procedure, starting with $b = 0$ and keep going until the condition $p_d \leq p_f(1 - F_f(C - b))$ is satisfied.
Furthermore, considering $F_f(C - b)$ is a cumulative distribution function, we obtain the following result:

Littlewood rule

The optimal number b^* to reserve at a lower fare is given by the resolution of the following equation:

$$1 - F_f(C - b^*) = \frac{p_d}{p_f}$$

Numerical example:
Supposing the request for the higher fare ticket is normally distributed:

$$F_f(x) = \Phi(\frac{x - \mu_f}{\sigma_f})$$

$$\mu_f = \text{valore atteso di } d_f \quad \sigma_f = \text{varianza di } d_f$$

Then we obtain that

$$b^* = [C - \sigma_f \Phi^{-1}(1 - \frac{p_d}{p_f}) - \mu_f]^+$$

If we suppose we have n classes, each of them with its fare p_i $i = 1, \cdot, n$ (supposing class 1 has the higher fare) and a request d_i $i = 1, \cdot, n$,the situation becomes complicated, because adjoin one seat at the i-esima class, we are removing it at one class with higher fare,so we need to introduce new random variables q_{ij} con $j > i$, representing the probability that, adjoing a seat at the i-esima class, we are removing it from jth class. This complicate a lot the situation.
The dinamic programming brings us a optimal solution but in a elevated timing.
As we said before, airlines need to have efficient instruments to calculate how reserve tickets with different fares. For this reason, they use heuristic techniques that give us a good (not optimal) results.

EMSR/ expected marginal seat revenue To calculate how many seats save for the jth, we create a fictitious class, in which request will be the sum of the expected request for the classes with a higher fare and the price will be the avarage between the price and then we will use Littlewood rule between jth class and the fictitious one.
Supposing requests are normally distributed with mean $\mu_i, \ i = 1, ..., n$ and variance $\sigma_i \ i = 1, ..., n$. Supposing to be in the periond $j \geq 2$ and we want to calculate b_j We create the fictitious class:

$$\mu = \sum_{i=1}^{j-1} \mu_i \quad \sigma = \sqrt{\sum_{i=1}^{j-1} \sigma_i^2} \quad p = \frac{\sum_{i=1}^{j-1} p_i \mu_i}{\mu}$$

Then we use i Littlewood rule

$$b_j = [C_j - \sigma \Phi^{-1}(\frac{p - p_j}{p}) - \mu]^+$$

there C_j represent the capacity of the plane when we are calculating the jth class, in othe words $C_j = C - \sum_{i=j+1}^{n} x_i$ where x_i are the accepted request for the ith class.

Ecample: Alitalia ROMA-JFK 9.00 – 12.55 **August 1**
Classica plus: Price $p_3 = 1916$ euro,mean $\mu_3 = 180$ people, variance $\sigma_3 = 30$;
Economy:Price $p_2 = 2229$ euro, mean $\mu_2 = 115$ people, variance $\sigma_2 = 25$;
Business: Price $p_1 = 3526$ euro, mean $\mu_1 = 55$ people, variance $\sigma_1 = 15$;
Airplane: BOING 777, capacity $C = 293$
Using *EMSR* technique, we build the fictitious class to calculate how many seats reserve fot the Classica plus:

$$\mu = \mu_2 + \mu_1 = 170, \quad \sigma = \sqrt{(\sigma_2)^2 + (\sigma_1)^2} = 40 \quad p = \frac{p_1\mu_1 + p_2\mu_2}{\mu} = 2649$$

dunque

$$b_3 = [C - \mu - \sigma\Phi^{-1}(\frac{p - p_3}{p})]^+ = 147$$

To calculate how many seats reserve for the Economy, we suppose to receive at least 147 request for the Classic plus, such that we will assign all the seats we have reseave. Updating our data:

$$\mu = \mu_1 \quad p = p_1 \quad \sigma = \sigma_1 \quad C = 293 - 147 = 146$$

$$b_2 = [C - \mu - \sigma\Phi^{-1}(\frac{p - p_2}{p})]^+ = 97$$

Then, the optimal solution with the hypothesis the people who book the lower fare, book before the others, is given by:
Reserve 147 seats for the Classica plus, 97 for the economy Economy, such that Business customers will have 49 seats.

Generalization

- We have considered so far only the situation in which the request for the classes are independent.We define now α the percentage of people who, not able to find ticket with a discount fare, they decide to buy at a higher fare. The request d_f for the ticket with the higher fare becomes:

$$\hat{d}_f = d_f + \alpha[d_b - b]^+$$

Doing the same calculation as before, we have that the Littlewood rules becomes find b^* such that

$$F_f(C - b^*) = 1 - (\frac{1}{1-\alpha})(\frac{p_d}{p_f} - \alpha)$$

- Sometimes the measurament *RAMS* is not the best one and it does not tell use complitely if we have done a good *RM* or not. An alternative is to use the *ROM* (revenue opportunity model) defined as:

$$ROM = \frac{\text{guadagno raggiunto} - \text{guadagno senza RM}}{\text{guadagno perfetto} - \text{guadagno senza RM}}$$

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Network Management

The model we have developed does not work anymore if we have multiple routes, for example connections flights. We can not think only about prices, but also about all possible combinations and expected revenue.
We make a simple example:
Flight1: Venezia-Roma, costo 200 euro
Flight2: Roma-Catania, costo 160 euro
Flight1+Flight 2: Venezia-Catania, costo 300 euro
We suppone we have only one seat free in each of two flight, and one person want to book a flight Venezia-Catania with stop over at Roma and so he want to pay us 300 euro. We will accept only if

$$200p_1 + 160p_2 < 300$$

where p_1 is the probability to receive a booking for the flight 1 and the same holds for p_2

A good approach, in the hypothesis to know the future requests for all the possible classes, is given by the **linear programming**. In fact

- It gives us an optimal solution;
- It can be used as a starting point for a more accurate solution, taking in consideration also the aleatority of the request.

In the real situation, airlines used some technique of stochastic linear programming or the "virtual nesting", where every product is mapped in a virtual class, that it takes in consideration not only the price and a booking is accepted only if there is availability in that class, availability calculated with previous methods.

Implementation

We consider m resources (capacity of the plane, for example) and n possible products. We index-link with i the resourse and with j the products. For each routes, we will have a price p_j and a deterministic request d_j, about which we will satisfie x_j. Every resourse has a limit c_i. Defining the following variables:

$$a_{ij} = \begin{cases} 1 & \text{if the product } j \text{ uses the resource } i \\ 0 & \text{altrimenti} \end{cases}$$

The linear programming problem becomes

$$\max \sum_{j=1}^{n} p_j x_j$$

$$\sum_{j=1}^{n} a_{ij} x_j \leq c_i \quad \forall i$$

$$0 \leq x_j \leq d_j \quad \forall j$$

Example Alitalia with 4 classes

Routes	Classes	Fare	Request	Allocation
Venezia-Catania	Promo	91.04	45	**3**
6.25-9.55	Facile	128.04	25	**25**
1 agosto	Comoda	302.04	10	**10**
100 posti	Libera	445.04	5	**5**
Venezia-Roma	Promo	51.84	25	**25**
6.25-7.30	Facile	107.81	18	**18**
1 agosto	Comoda	333.81	8	**8**
100 posti	Libera	428.81	4	**4**
Roma-Catania	Promo	101.40	30	**30**
8.30-9.55	Facile	122.40	15	**15**
1 Agosto	Comoda	297.40	9	**9**
100 posti	Libera	417.40	3	**3**

Overbooking

The overbooking consists to sell more units of which you have available

- Up to 1990 there was not a fee, in the case of cancelation of the booking or no-show;
- It has been extimated that, even with the sold-out, AA traveled with 15% of empty seats and the 50% of booking was later canceled.
- Autorized to overbook to protect from possible losses, AA has extimated a revenue of 225 $ milion on 1990 thanks to overbooking.
- Despite all airlines now apply a fee if you want to cancel a booking, they still continue doing overbooking to maximize the revenue.

In the case the airline make wrong calculations it is forced to forbit a passenger with regular ticket to get inside the plane. It has arised regulation also in this case.

How to calculate the allowed number of seats over the capacity in order to maximize revenue? There are four technique utilised:

1. **Heuristic method**: calculate a booking limit based on the capacity and the expected no-show
2. **Risk method**: Explicit extimation of denied servig, comparing this calculation with the expected potential revenue.
3. **Level method**: For example, we allowed a denied service every tot passengers served
4. **Hybrid method**: A method base on the union of the three previous methods. Un metodo basato sull'unione dei tre metodi precedenti.

Description of the model

- Fixed capacity C
- Maximum limit of booking b (clearly $b \geq C$)
- The day of the flight, the customers pay a price p for the ticket.
- If the number of the customers isi C then everyone can find a seat, otherwise only C will be allowed to get into the plane and $b - C$ will receive a compensation D ($D > p$).

We suppose there are not cancelation, otherwise b should change over time.

Problem

Determinate b in order to maximize the expected revenue.

Heuristic method

An arline have calculated that the rate of people who show is ρ. Then, it fix $b = \frac{C}{\rho}$. A lot of airlines still use this method

Risk method

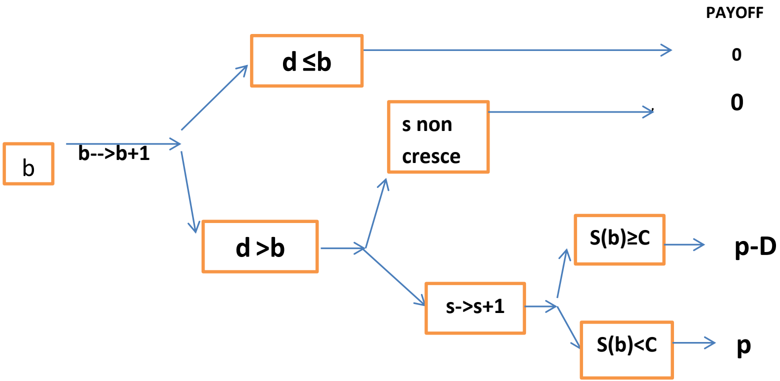
Let d be the total requesr received:

$$n(b) = \min(b, d) \quad x(b) = n(b) - s(n(b))$$

where $s(n(b))$ is the number of people who show up. Then $x(b)$ is the number of people who doen not show up. Let $\rho = \frac{s(n(b))}{n(b)}$ be the rate of people who show up. The expected revenue for this route is given by

$$R = p * s(n(b)) - D * [s(n(b)) - C]^{+}$$

We suppose we decide a limit b of booking. What happen if we raise it at $b + 1$?



If the expected value of this operation is positive, then raise the level makes sense. Doing the calculation

$$E[h(b)] = [1 - F(b)]\rho[(p - D)P(s(b) \geq C) + p(1 - P(s(b) \geq C) =$$

$$= \rho[1 - F(b)][p - DP(s(b) \geq C)]$$

where $P(s(b) \geq C)$ is the probability that, fixed b maximum booking, the number of people who show up is greater or equal than C .
As, if $b$raises also $s(b)$ raise and then also the probability, we end up with

Rule

The optimal maximum limit of booking b^* has to satisfy at the following

$$P(s(b^*) > C) = \frac{p}{D}$$

Under the hypothesis each person show up with a rate ρ independent from the others, we have that $s(n(b))$ has a binomial distribution

$$P(s(n(b)) = x) = \binom{n(b)}{x} \rho^x (1-\rho)^{n(b)-x}$$

Moreover

$$P(s(n(C)) \geq C) = [1 - F(C-1)]\rho^C$$

In fact, fixed a limit of C booking, the probability that people who show up are not less than C is given by the fact that we have to have at least C booking and that all the people who book have to show up. e che tutte le persone che si prenotano si presentino. Furthermore we have that

$$P(s(n(b+1)) \geq C) - P(s(n(b)) \geq C) = \rho[1 - F(b)]P(s(b) = C-1)$$

in fact, I have to have that:

1. number of booking is at least b
2. the adjoint seat we add, belongs to a person that show up
3. fixed the limit of b booking, exactly $C-1$ people show up.

For what we have seen in the previous slide

$$P(s(b) = C - 1) = \binom{b}{C-1}\rho^{C-1}(1-\rho)^{b-C+1}$$

We have the following method:

Algorithm for the optimal limit of booking

1. Set $b = C$ e $P(s(n(b)) \geq C) = [1 - F(C)]\rho^C$
2. If $P(s(n(b)) \geq C) = \frac{p}{D}$ puti $b^* = b$ and stop.
3. Otherwise, $b- > b + 1$ and $P(s(n(b+1)) \geq C) =$

$$P(s(n(b)) \geq C) + [1 - F(b+1)]\binom{n}{C-1}\rho^C(1-\rho)^{b-C+1}$$

and come back to 2.

Level method We notice that $P(s(n(b)) \geq C+1)$ is the probability that, set a maximum limit b of booking, at least $C+1$ people show up and then at least one person has to be denied from the boarding. Then, if the airline want to gain without demage the customers, then it could choose b^*, such that, fixed ϵ,

$$P(s(n(b^*)) \geq C+1) \leq \epsilon$$

Overbooking with more classes In the case of different fares, a strategy that can be used is to consider the avarage price, weigh with the average request for each classes: $\hat{p} = \frac{\sum_i \mu_i p_i}{\sum_i \mu_i}$ and then find the maximum limit b of booking. Then, how many seats reserve for each classes can be done as previous section, considering b and not C as capacity.

Alternatives to overbook

The overbooking can bring the airline to a good revenue but it could also be damage for it. There are different approaches can be used:

- "Standby booking", in which it sells the tickets at a lower price, but you have the guarantee of the seat only when you arrive to the airport
- If it has a lot of request, above all in the high fare classes, airline can call the customers who have already booked and offer a compensation for take the next flight.
- There are fee (up tp100%) in the case of cancelation and no-show.

Conclusion

- At the bottom of RM , we need an optimal analysis of historical data and prevision of the future request.
- An optimal RM can give the airline a raise 5/8% of revenue
- Can be generalized also to hotel, rental cars, cruises, theater
- There are a lot of heuristic techniques: it is a open problem to find models that give optimal results in a short timing.

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